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LETTER TO THE EDITOR

Fermionic Goldstone–Higgs effect in (2+1)-dimensional supergravity

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Abstract. The non-linear spinor realisation of global supersymmetry is consistently coupled to supergravity in (2+1) dimensions. The resulting local invariance permits gauging away of the spinor field, while giving an effective mass to the spin-$\frac{3}{2}$ component of supergravity. The latter was non-dynamical when massless, but with mass it corresponds to a massive spin-$\frac{1}{2}$ excitation.

1. Introduction

In a previous letter (Dereli and Deser 1977a, b) whose notation we follow, a two-dimensional model for the coupling of non-linear realisations of global supersymmetry (Volkov and Akulov 1973) to supergravity was given. The model displayed the features expected in the full four-dimensional case (Deser and Zumino 1977), where the corresponding coupling has not yet been completely obtained. However, the two-dimensional model displayed a peculiar discontinuity owing to the identical vanishing of the supergravity action. The original spinor degree of freedom, when coupled to supergravity, was 'swallowed' by the spin-$\frac{3}{2}$ field, but the latter did not acquire any dynamics in the process. In this paper, we consider the three-dimensional case, which does have a non-trivial (though non-dynamical) supergravity action (Howe and Tucker 1978), and where the consequent absorption of the spinor by the spin-$\frac{3}{2}$ field will exhibit the expected fermionic Higgs behaviour: the fermion's degree of freedom is transferred to the spin-$\frac{3}{2}$ field, even though the latter (like massless spin-1 in two dimensions) originally had no dynamics in (2+1) dimensions. This is due to the fact that massive spin-$\frac{3}{2}$ does have the dynamical content of one lower unit of spin (again like massive spin-1 in two dimensions), and that (as in four dimensions), an apparently massless spin-$\frac{3}{2}$ field in a background de Sitter (rather than Minkowski) space is effectively massive. (The link between the latter two statements is the cosmological term left over from the fermion's action after its field has been gauged away.) Thus three dimensions display the desired features of four, while enjoying those simplifications of two, dimensions which make possible an explicit formulation.

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2. The model

In $(2+1)$ dimensions, spinors have two components and the Dirac matrices reduce to the Pauli σ^i (with real σ^2), with $[\gamma^\mu, \gamma^\nu] = 2\epsilon^{\mu\nu\alpha}\gamma_\alpha$. The flat space fermion action†

$$I_{\mathrm{M}} = \frac{1}{2a^2}\int \mathrm{d}^3x \ \det[\delta_\mu{}^a - \mathrm{i}a^2\bar{\lambda}\gamma^a\partial_\mu\lambda] \tag{1}$$

is invariant under the non-linear transformation

$$\delta\lambda = a^{-1}\alpha - \mathrm{i}a(\bar{\alpha}\gamma^\mu\lambda)\partial_\mu\lambda \tag{2}$$

where α is a constant spinor.

We shall couple (1) to supergravity as in two dimensions by the covariant substitutions

$$\delta_\mu{}^a \to e_\mu{}^a, \qquad \partial_\mu\lambda \to (\mathrm{D}_\mu\lambda - a^{-1}\kappa\psi_\mu). \tag{3}$$

Here $e_\mu{}^a$ is the *vierbein* field, D_μ the covariant derivative on a spinor,

$$\mathrm{D}_\mu\lambda \equiv (\partial_\mu + \tfrac{1}{2}\omega_\mu{}^a\gamma_a)\lambda, \tag{4}$$

with $\omega_\mu{}^a \equiv \frac{1}{2}\epsilon^{abc}\omega_{\mu bc}$ the dual of the usual connection, while ψ_μ is the spin-$\frac{3}{2}$ field and κ is the gravitational constant (a and κ are independent dimensional constants). Thus we obtain for the matter action,

$$I_{\mathrm{M}}(\lambda; e, \psi, \omega) = \frac{1}{2a^2}\int \mathrm{d}^3x \ \det[e_\mu{}^a - \mathrm{i}a^2\bar{\lambda}\gamma^a(\mathrm{D}_\mu\lambda - a^{-1}\kappa\psi_\mu)]. \tag{5}$$

The (e, ψ) couple to the stress tensor and supercurrent respectively; the flat space form of the latter is $J^\mu \sim -\gamma^\mu\lambda + \mathrm{i}a^2\bar{\lambda}\lambda\epsilon^{\mu\nu\rho}\gamma_\nu\partial_\rho\lambda$. In coupling I_{M} to the gauge fields $(e_\mu{}^a, \omega_\mu{}^a, \psi_\mu)$, we promote the global invariance to a local one, $\alpha \to \alpha(x)$; the following transformations:

$$\begin{aligned} &\delta\lambda = a^{-1}\alpha(x), \qquad \delta\psi_\mu = 2\kappa^{-1}\mathrm{D}_\mu\alpha(x), \qquad \delta\omega_\mu{}^a = 0 \\ &\delta e_\mu{}^a = \mathrm{i}\kappa\bar{\alpha}\gamma^a\psi_\mu + \mathrm{D}_\mu\xi^a, \qquad \xi^a \equiv \mathrm{i}a\bar{\alpha}\gamma^a\lambda \end{aligned} \tag{6}$$

leave (5) invariant. Note that we have apparently linearised the λ transformation law in going from (2) to (5); however, the original form could be recovered by noting that the $\mathrm{D}_\mu\xi^a$ part of δe is a coordinate transformation, and can therefore, by coordinate invariance of the total action (5), be transferred back to $\delta\lambda$ (and to the other variables) if desired.

The next step is to consider the effect of the transformation (6) on the gauge field (supergravity) action, whose properties in $(2+1)$ dimensions have been discussed by Howe and Tucker. Note that the (e, ψ) variations are already the usual ones for supergravity, except for the coordinate transformation in δe. The vanishing of $\delta\omega$ will also turn out to be correct here due to the peculiarities of three dimensions.

In first-order form we may write the supergravity action I_{SG} as follows

$$I_{\mathrm{SG}} = I_2 + I_{3/2} = \kappa^{-2}\int \mathrm{d}^3x \ {}^{**}R^{\mu a}(\omega)e_{\mu a} - \mathrm{i}\int \mathrm{d}^3x\bar{\psi}_\mu{}^*f^\mu \tag{7}$$

$${}^{**}R^{\mu a} \equiv \tfrac{1}{4}\epsilon^{\mu\nu\sigma}\epsilon^{abc}R_{\nu\sigma bc}(\omega); \qquad {}^*f^\mu \equiv \tfrac{1}{2}\epsilon^{\mu\nu\sigma}\mathrm{D}_\nu\psi_\sigma. \tag{8}$$

† Note that because all higher than quadratic powers of the two-component λ vanish, the cubic term in $(\bar{\lambda}\gamma\partial\lambda)$ is really absent from (1) and only linear $\omega\bar{\lambda}\lambda$ dependence occurs in (5).

Note that, unlike the four-dimensional case, the double dual $^{**}R^{\mu a}$ of the curvature is a vector density and so I_2 is linear in $e_{\mu a}$. The Rarita–Schwinger action is totally independent of $e_{\mu a}$, so its (non-symmetric) stress tensor vanishes, and I_{SG} therefore describes a flat space (since $^{**}R^{\mu a}=0$ implies $R_{\mu\nu ab}=0$) but with torsion. The spin-$\frac{3}{2}$ field strength (defined in terms of the torsion) likewise vanishes. The invariances of I_{SG} are much simpler than in four dimensions: it is easy to see that under

$$\delta\psi_\mu = 2\kappa^{-1}\mathrm{D}_\mu\alpha, \qquad \delta e_\mu{}^a = -\mathrm{i}\kappa\bar{\alpha}\gamma^a\psi_\mu \tag{9}$$

the gauge field action (7) is invariant without help from $\delta\omega_\mu{}^a$, so that we set $\delta\omega_\mu{}^a=0$, even in first-order† form.

It is now almost trivial to verify the invariance of the matter–supergravity action $I_{\mathrm{M}}+I_{\mathrm{SG}}$ of (5) and (7) under (6) which already leaves I_{M} invariant. The only relevant difference in the transformations (6) and (9) lies in the extra term in $\delta e_\mu{}^a$ of (6); this term can only affect I_2 since $I_{3/2}$ is independent of e. But the contribution from I_2 vanishes as a result of the Bianchi identity, $\mathrm{D}_\mu(\omega)^{**}R^{\mu a}(\omega)\equiv 0$, which holds also for arbitrary non-metric ω; as a result,

$$\delta_\xi I_2 \equiv \int \mathrm{d}^3x\,\mathrm{D}_\mu\xi_a{}^{**}R^{\mu a} = -\int \mathrm{d}^3x\,\xi_a\mathrm{D}_\mu{}^{**}R^{\mu a}\equiv 0 \tag{10}$$

which completes the proof of invariance of the combined action.

3. The super-Higgs effect

Having obtained the coupled locally supersymmetric action, we may now use the gauge freedom $\delta\lambda = a^{-1}\alpha(x)$ to eliminate λ. In this gauge, the matter action reduces to the term $(1/2a^2)\det e$, so the total action is just pure supergravity plus a cosmological term. This system would still seem to be non-dynamical, particularly in the $\frac{3}{2}$ sector which is independent of e, and the λ degree of freedom appears to have been lost as in the $(1+1)$ case. However, we now see that, just as in the four-dimensional analysis (Deser and Zumino 1977), the spin-$\frac{3}{2}$ action in the presence of a cosmological term really describes a massive field, and that as a result it *does* have dynamics equivalent to a spin-$\frac{1}{2}$ field, in complete parallel with massive and massless electrodynamics in $(1+1)$ dimensions‡.

Let us begin with the free massive spin-$\frac{3}{2}$ case. The appropriate mass term is $-\frac{1}{2}\mathrm{i}m\epsilon^{\mu\nu\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\sigma$ and the field equation reads

$$f_{\mu\nu}+m(\gamma_\mu\psi_\nu-\gamma_\nu\psi_\mu)=0, \qquad f_{\mu\nu}\equiv\partial_\mu\psi_\nu-\partial_\nu\psi_\mu. \tag{11}$$

Thus $m=0$ corresponds to vanishing dynamics; if $m\neq 0$, we learn from taking the divergence and γ^μ-contraction of (11) that $\gamma\,.\,\psi=0=\partial^\mu\psi_\mu$, as expected, and (11) reduces to

$$(\partial\!\!\!/+m)\psi_\mu=0, \qquad \partial\,.\,\psi=0=\gamma\,.\,\psi. \tag{12}$$

† If one computes $\delta\omega(e,\psi)$ in second-order form it also vanishes (modulo the Rarita–Schwinger equation), as required for consistency.

‡ For the vector field, since there is only one field strength F_{01}, the Maxwell equations state that it vanishes and A_μ is pure gauge for $m=0$. If, however, $m\neq 0$, then we first learn from the Proca equation that $\partial^\mu A_\mu=0$ as usual. This means that $A^\mu=\epsilon^{\mu\nu}\partial_\nu\phi$, in terms of which the Proca equation reduces to the Klein–Gordon equation for the scalar ϕ.

The two auxiliary conditions reduce the three $(\mu = 0, 1, 2)\psi_\mu$ to a single spinor χ which satisfies the Dirac equation and is dynamical. These features may also be made explicit by the Hamiltonian analysis of the system (Deser *et al* 1977), in which χ is expressed as an appropriate component of $\psi_i (i = 1, 2)$. The action reads

$$I_{3/2}(m) = -\tfrac{1}{2}\mathrm{i} \int \epsilon^{ij}\bar{\psi}_i(\partial_0 + m\gamma_0)\psi_j \, \mathrm{d}^3x \tag{13}$$

with the constraint $\epsilon^{ij}(\partial_i + m\gamma_i)\psi_j = 0$ obtained by varying with respect to ψ_0. Decomposing ψ_i into curl and gradient, $\psi_i = \epsilon^{ij}\partial_j\eta + \partial_i\phi$ leads to the relation $(\nabla\!\!\!/ + m)\eta = m\gamma^0\phi$. When inserted into (13) the latter reads

$$I_{3/2}(m) = -\frac{\mathrm{i}}{m} \int \bar{\eta}_{,i}(\gamma^0\partial_0 + m)(\nabla\!\!\!/ + m)\eta_{,i}.$$

The $\gamma^0\partial_0\nabla\!\!\!/$ part is a total divergence, leaving finally the massive form:

$$I_{3/2}(m) = -\tfrac{1}{2}\mathrm{i} \int \mathrm{d}^3x \, \bar{\chi}(\partial\!\!\!/ + m)\chi, \qquad \chi \equiv \sqrt{-2\nabla^2}\,\eta. \tag{14}$$

Next, we show that the apparently massless theory in a constant curvature background corresponds to the above massive description; since Minkowski space is not a solution in the presence of a cosmological term, constant curvature is the corresponding 'flattest' background satisfying the cosmological Einstein equations. In this (torsion-free) geometry, the curvature is just

$$R_{\mu\nu}{}^{bc} = \Lambda(e_\mu{}^b e_\nu{}^c - e_\mu{}^c e_\nu{}^b) \tag{15}$$

and therefore

$$\frac{2}{\Lambda}[\mathrm{D}_\mu, \mathrm{D}_\nu]\alpha = e\epsilon_{\mu\nu\rho}\gamma^\rho\alpha \tag{16}$$

for the commutator of covariant derivatives on any spinor. Thus taking the covariant divergence of the Rarita–Schwinger equation $^*f^\mu = 0$ immediately implies that

$$\gamma \cdot \psi = 0. \tag{17}$$

Using $\gamma \cdot {}^*f = 0$, one then sees that $\partial^\mu\psi_\mu$ is also determined. The analysis is similar to that of Deser and Zumino, and shows that this system is equivalent to a single massive spinor field in the background space. The λ degree of freedom has therefore been transferred to the ψ_μ gauge field through this mechanism precisely as expected in four dimensions†.

Our three-dimensional model thus confirms the four-dimensional expectations in detail. In addition the simple form of the field transformations used here in which $\delta\lambda$ is linearised at the price of a coordinate transformation in $\delta e_\mu{}^a$, should provide a useful start for the four-dimensional problem.

† In this connection, it would be interesting to couple (1) to the version of (7) in which additional matched mass and cosmological terms are present. Cancellation between the two cosmological constants would then leave a net ψ_μ mass term.

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